Assignment 2

Graph Searching and Shortest Paths

COMP-362 Winter Term 2004

1. Modify the depth-first search algorithm so that given a digraph G as its input it will determine whether or not there are directed cycles in G.

2. For a digraph $G = (V, E)$ we define its *adjacency matrix* $A(G)$ to be the matrix whose columns and rows are indexed by the set V and the uv entry, where $u, v \in V$, is equal to 1 if $(u, v) \in E$ - otherwise it is equal to 0.

   (a) What is the uv entry of $A^k(G)$ for $k \geq 1$ and any $u, v \in V$?

   (b) Give an algorithm that determines the length of the smallest cycle of G and the number of cycles having this length.

3. Let $G = (V, E)$ be a weighted digraph with $w : E \to \mathbb{R}^+$ being its weight function. If $p = (v_0, \ldots, v_k)$ is a directed path, we define its weight to be $w(p) = \min\{w(v_i, v_{i+1})\}_{i=0,\ldots,k-1}$. Let $s \in V$ be a reference vertex. We define the *capacity* of a vertex $v \in V \setminus s$, which we denote $c(v)$, as $c(v) = \max\{w(p) \ : \ p \text{ is a path from } s \text{ to } v\}$ - if there is no path from s to v, then we set $c(v) = 0$. Design a dynamic programming algorithm that determines the capacities of the vertices in $V \setminus s$.

Due date: 17th February 2004