COMP-362: Honours Algorithm Design

Assignment 4: Linear Programming and Complexity Theory

Winter Term 2004

1. Consider a linear program: $\max c^T x$ subject to $Ax \leq b$, $x \geq 0$ (we call it (LP1)).
   (a) Prove that if there is $0 \leq y \in \mathbb{R}^m$ with $A^T y \geq 0$ and $b^T y < 0$, then (LP1) has no solution.
   (b) Prove that if (LP1) has a feasible solution and there exists $z \geq 0$ such that $Az \leq 0$ and $c^T z > 0$, then (LP1) is unbounded.
2. Consider a linear program: $\max c^T x$ subject to $Ax \leq b$, $x \geq 0$ (again we call it (LP1)). Prove that the dual of the dual is (LP1) itself (up to equivalence).
3. Suppose that a language L is decided by a non-deterministic Turing machine within time $f(n)$, for some function $f : \mathbb{N} \to \mathbb{N}$. Describe a *deterministic* Turing machine that decides L and give a bound on the time this machine needs (up to multiplicative constants). (Do not describe the transition function in detail; just describe how you use the non-deterministic Turing machine and, in particular, its transition relation.)
4. Prove that $\text{NTIME}(f(n)) \subseteq \text{SPACE}(f(n))$, where $f : \mathbb{N} \to \mathbb{N}$. (Again if you need to describe a Turing machine, do it in rough terms.)

Due date: 30th of March 2004.